show the variation of $P(T)$ with T, but any rigorous calculation of the equation requires the values of Q which is not known for many molecules. The objective of the present investigation has been, however, to develop an expression which could be applied to explain the anomalous behavior in certain polar molecules.

Equilibria of Weak Complexes by Solution Calorimetry

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Abstract: Expressions are developed for determining thermodynamic parameters of several simple reaction mechanisms from the combination of two types of solution calorimetry experiments, heats of infinite dilution and heats of infinitesimal dilution. The advantages and limitations of combining these measurements are discussed. Application of the method is given for the cases of urea self-association and for purine self-association. In both cases it is shown that the thermodynamic quantities derived from the two types of heat measurements agree with those obtained from studies over a range of concentrations. The advantage of the procedure is the determination of weak interaction parameters on relatively dilute solutions.

The study of relatively weak hydrogen-bonding or charge-transfer complexes has relied mainly upon spectroscopic techniques.[2-4] Limited solubility or complications due to spectral absorption by the solvent can restrict their application. In those cases the methods of solution thermodynamics may be used.[2,4] The nonideal behavior, as measured by activity coefficients,[5,6] osmotic coefficients, or apparent molecular weights by sedimentation equilibria,[7] is interpreted in the context of various simple reactions. Under favorable circumstances, temperature derivatives of the evaluated free energies have been used to determine heats and entropies of reactions.

A less familiar approach has made use of mixing calorimetry.[8-10] In some cases sufficient information can be obtained to determine ΔG°, ΔH°, and ΔS° without auxiliary information; in other cases the combination of heats of dilution with osmotic coefficient data provides a route for ΔH° evaluations.[11]

In this paper we want to show how two types of solution calorimetry experiments, heats of infinite dilution and heats of infinitesimal dilution, can be used for the study of certain simple reactions involving weak interactions.

Heats of Infinite Dilution

When a solution containing m moles of solute and 1 kg of solvent is diluted by an infinite amount of solvent, the heat of this dilution is expressed by $-m\varphi_L$. The quantity φ_L is the relative apparent heat content. We assume this heat effect is due entirely to the dissociation of complex species into monomeric forms. Equilibria conditions govern the concentration of various species in the solution of molality m. For, example, self-association reactions might occur of the form with appropriate equilibrium constants and enthalpies of reaction

$$\begin{aligned} &\mathrm{A} + \mathrm{A} \rightleftarrows \mathrm{A}_2 \qquad K_2,\ \Delta H_2^\circ \\ &\mathrm{A} + \mathrm{A}_2 \rightleftarrows \mathrm{A}_3 \qquad K_3,\ \Delta H_3^\circ\text{, etc.} \end{aligned} \tag{1}$$

In general, insufficient precision of measurements or incomplete validity of describing the nonideal behavior by such schemes precludes evaluation of more than one or two constants.

The heat of infinite dilution, $-m\varphi_L$, can be written formally in terms of molal concentrations as

$$m\varphi_L = (\mathrm{A}_2)\Delta H_2^\circ + (\mathrm{A}_3)(\Delta H_2^\circ + \Delta H_3^\circ) + \cdots \tag{2}$$

or

$$m\varphi_L = K_2(\mathrm{A})^2\Delta H_2^\circ + K_2K_3(\mathrm{A})^3(\Delta H_2^\circ + \Delta H_3^\circ) + \cdots$$

If all ΔH_n° values are equal to ΔH°, then a simpler result[9-11] can be obtained with the definitions of the osmotic coefficient Φ

$$\varphi_L = (1 - \Phi)\Delta H^\circ \tag{3}$$

This equation is applicable to any variety of reactions described by (1) for equal ΔH° values. Relative osmotic coefficients could be obtained by applying this equation in reverse.

If more restrictive conditions are imposed within eq 1, two special cases show similar concentration dependence of φ_L. For a dimerization reaction

$$\varphi_L = \frac{\Delta H_2^\circ}{2} - {}^1/_2\left(\frac{\Delta H_2^\circ}{K}\right)^{1/2}\left(\frac{\varphi_L}{m}\right)^{1/2} \tag{4}$$

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and in a polymerization case of unlimited species with equal K's and $\Delta H°$'s

$$\varphi_L = \Delta H° - \left(\frac{\Delta H°}{K}\right)^{1/2}\left(\frac{\varphi_L}{m}\right)^{1/2} \qquad (5)$$

We find for the general case $n(A) = (A)_n$ that

$$\varphi_L = \frac{\Delta H_n°}{n} - \frac{\Delta H_n°}{nm}\left(\frac{m\varphi_L}{K\Delta H_n°}\right)^{1/n} \qquad (6)$$

which reduces to the above dimerization case when $n = 2$. It is also possible to solve the m dependence explicitly for φ_L in terms of ΔH_n and K. For example, eq 5 gives

$$\varphi_L = \Delta H°Km[1 - 2Km + 3K^2m^2 - 6K^3m^3 + \cdots] \qquad (7)$$

for the polymerization case of unlimited species. Heats of dilution data on urea by Gucker and Pickard[12] follow this form, and the evaluated K (0.04 m^{-1}) and $\Delta H°$ (−2160 cal) agree well with values obtained by Schellman[11] (0.041 m^{-1} and −2190 cal) using a combined approach such as that given in eq 3 or those obtained by Kresheck[13] and Scheraga (0.041 m^{-1} and −2160 cal) from the temperature dependence of nonideality of urea solutions.

The form of eq 4 and 5 shows that heats of infinite dilution alone cannot distinguish between a dimerization or a multiple polymerization case; auxiliary information must be used. The situations described by eq 6 when $n \neq 2$ can be tested for linearity for various n values.

The practical aspect of applying eq 4 or 5 requires sufficient complex formation to define the intercepts of $\Delta H_2°/2$ or $\Delta H°$. In either case for low amounts of complex formation $\varphi_L \cong mK\Delta H°$ and the heat of infinite dilution is proportional to the combination $K\Delta H°$. This result suggests that if heats of dilution studies are carried out at at least two different temperatures and the temperature dependence of K is expressed in terms of $\Delta H°$, then both $\Delta H°$ and K can be determined. A decision of the applicable reaction equation can then be made from eq 4 or 5 using the determined $\Delta H°$.

Heat of Infinitesimal Dilution

The determination of the heat of infinitesimal dilution may also be used to study the thermodynamic properties of a solution. The addition of a small amount of solvent to a large amount of solution at a given concentration describes a differential heat of solution, which for a mole of solvent gives the heat effect

$$\Delta H = \bar{H}_1 - \bar{H}_1° = \bar{L}_1 \qquad (8)$$

where $\bar{H}_1$ is the partial molal enthalpy of solvent in solution, $\bar{H}_1°$ refers to the pure state, and $\bar{L}_1$ is the relative partial molal enthalpies of solvent.

Thermodynamic relations[14] between φ_L and the relative partial molal enthalpies of solvent $\bar{L}_1$ and solute $\bar{L}_2$ are given by

$$\bar{L}_2 = \frac{\partial}{\partial n_2}(n_2\varphi_L) \qquad (9)$$

$$\bar{L}_1 = -\frac{n_2^2}{n_1}\left(\frac{\partial \varphi_L}{\partial n_2}\right)_{n_1} \qquad (10)$$

For an aqueous m molal solution with 55.5 moles of solvent water

$$\bar{L}_1 = -\frac{m^2}{55.5}\left(\frac{\partial \varphi_L}{\partial m}\right) \qquad (11)$$

For the case of monomer–n-mer equilibria, using eq 6

$$\bar{L}_1 = \frac{-m}{55.5}\frac{\Delta H_n° - n\varphi_L}{\Delta H_n° + (n^2 - n)\varphi_L}(n - 1)\varphi_L \qquad (12)$$

or if $n = 2$

$$\bar{L}_1 = \frac{-m}{55.5}\frac{\Delta H_2° - 2\varphi_L}{\Delta H_2° + 2\varphi_L}\varphi_L \qquad (13)$$

for the case of multiple species with eq 5

$$\bar{L}_1 = \frac{-m}{55.5}\frac{\Delta H° - \varphi_L}{\Delta H° + \varphi_L}\varphi_L \qquad (14)$$

Examination of these situations again reveals that measurements on $\bar{L}_1$ and φ_L alone cannot distinguish between the cases of dimerization and multiple species. A test on the general adequacy of the equation is available from measurements over a range of concentration. Measurements for $\bar{L}_1$ and φ_L would be needed for a solution of a given concentration. Application of these equations then avoids the sometimes tenuous extrapolation introduced by eq 5 and 6. Once $\Delta H°$ or $\Delta H_n°$ is known, K can be calculated from either eq 5 or 6.

An idea of the limitations of general applicability of these procedures is shown by examining the low concentration behavior of $\bar{L}_1$ starting from the general description of eq 2 and the equation for molality m

$$m = (\mathrm{A}) + 2K_2(\mathrm{A})^2 + 3K_2K_3(\mathrm{A})^3 + \cdots \qquad (15)$$

When (A) is expressed in terms of m and eq 11 is used, we find for the first two terms

$$\bar{L}_1 = -\frac{K_2\Delta H_2°m^2}{55.5} + \frac{8K_2^2\Delta H_2° - 2K_2K_3(\Delta H_2° + \Delta H_3°)}{55.5}m_3 + \cdots \qquad (16)$$

This equation shows that a determination of the coefficients of m^2, m^3, etc. will in general yield fairly complex combinations of equlibrium constants and enthalpies of reaction.

When it is known that one of the simple mechanisms used for eq 4–6 is valid, then eq 12, 13, or 14 can be used without resorting to series expansions. We shall apply this approach to two cases where sufficient information is available. In both situations multiple species related by equal equilibrium constants apply and so eq 14 will be used.

Gucker and Pickard[12] studied the dilution of urea at 25°. In their experiments either a small amount of solution was added to a larger amount of solvent or *vice versa*. This procedure yields φ_L and $\bar{L}_1$ to a good approximation. Their data are presented as a smooth function of the molality

$$\varphi_L = -85.87m + 6.815m^2 - 0.4569m^2 + 0.0147m^4 + \cdots$$

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Table I shows the evaluated values of φ_L and $\bar{L}_1$ for a selected set of molalities along with the $\Delta H°$ values computed by means of eq 14. The agreement with the previous methods is unexpectedly good in view of the wide range in concentration. The agreement adds further confirmation of the assumed polymer species reaction.

Table I. Enthalpies of Urea Solutions (25°)

Molality	φ_L, cal mole^{-1}	$\bar{L}_1$, cal mole^{-1}	$\Delta H°$, cal (eq 14)
0.20	−16.91	0.0599	−2096
0.25	−21.05	0.0929	−2105
0.50	−41.29	0.3573	−2076
1.00	−79.50	1.324	−2041
2.00	−147.90	4.58	−1966
4.00	−259.91	14.26	−1923
8.00	−424.48	39.67	−1995
		Av	−2020 ± 70

A second example is purine in water. This material forms complexes, which obey the polymerization case. Earlier experiments had been run to determine φ_L.[9] A direct determination of $\bar{L}_1$ for a 0.505 *m* solution was made. Values of heat per mole of solvent addition L_1(appl) are given in Table II for various dilutions. The extrapolated value, *i.e.*, $\bar{L}_1$, for an infinitesimal dilution is 5.73 cal mole^{-1}. The values of φ_L at various concentrations provide a determination of φ_L of −1690 cal m^{-1} at 0.505 *m*.[9] Application of eq 11 gives $\Delta H°$ of 3701 cal and with eq 5 *K* is 3.06 m^{-1}. These values are in essential agreement with the previous evaluations[9] ($\Delta H° = -3.7 \pm 0.2$ kcal/mole and $K = 2.9 \pm 0.2\ m^{-1}$) using the dependence of heats of infinite dilution on concentration.

Table II. Heat of Solvent Addition to 0.505 *m* Purine (25°)

Final concn, *m*	L_1(appl), cal mole^{-1}
(10/11) × 0.505	5.44
(15/16) × 0.505	5.56
(30/31) × 0.505	5.63
(1 × 0.505	5.73) extrapolated

Discussion

The suggested determination of the thermodynamic factors of weak complexes by solution calorimetry is borne out in the two cases presented above. The method can be expected to work for dimerization as well. Where *K* values are large, φ_L is experimentally difficult to determine because of the low concentrations needed to achieve reasonable fractions of dissociation. At such low concentrations $\bar{L}_1$ is also small. Conversely, small values of *K* make the determination of φ_L and $\bar{L}_1$ uncertain unless a concentration can be used where there is a reasonable fraction of complex species. The high sensitivity of the calorimetric measurement makes it possible to determine a fairly wide range of *K* and $\Delta H°$ values. One assigns the heat effect to the presence of complex species in much the same way as one uses a new spectral band to characterize a given complex. Thus the presence of large monomer species concentrations has no effect, whereas in an examination of colligative properties the effect of the monomer is predominant at low fractions of complex formation.

The general range of determinable *K* and $\Delta H°$ values depends upon several factors. Consider the situation for unlimited species complexes. As already noted the quantity $-m\varphi_L$ is the heat of diluting an *m* molal solution solution containing *m* moles of solute and 1 kg of solvent to infinite dilution. When the solution is sufficiently dilute ($Km < {}^1/_4$) such that φ_L is a small fraction of $\Delta H°$, then by eq 5 $-m\varphi_L \cong m^2K\Delta H°$. Thus the heat effect falls off quite markedly with increasing dilution, and a dilution by a factor of 10 will provide conditions where φ_L is essentially determined by a single experiment. For example, when 10 kg of solvent is added to the *m* molal solution, with a heat capacity of 10^4 cal deg^{-1}, 1-cal effects can be measured to 1% with temperature detection sensitivity of 1 μdeg. With this experimental limit $m^2K\Delta H° \approx 1$. For a complex where $\Delta H° = 1000$ cal, we can expect measurable heat effects for *K* values for various solution concentrations as follows: $K = 10^{-3}\ m^{-1}$ ($m = 1$ molal); $K = 10^{-1}\ m^{-1}$ ($m = 0.1$ molal); $K = 10\ m^{-1}$ ($m = 0.01$ molal). All of these situations meet the dissociation requirement of $Km < {}^1/_4$.

Similar practical limits arise in the determination of $\bar{L}_1$. With calorimetric technique of microdegree sensitivity $\bar{L}_1 \geq M_1/1000$ for 1% precision. The precision of measuring small values of $\bar{L}_1$ is comparable to the precision of measuring $-m\varphi_L$ as may be seen from eq 14.

The use of eq 14 to determine *K* and $\Delta H°$ values depends on the limiting concentration dependence of φ_L. With the approximation $\varphi_L \simeq -mK\Delta H°$, eq 14 is

$$L_1\,55.5 \cong m^2K\Delta H° \left(\frac{1 + mK}{1 - mK}\right) \qquad (17)$$

or

$$L_1 \frac{55.5}{m^2} \cong K\Delta H° \,(1 + 2\,mK + \cdots) \qquad (18)$$

where $mK \ll 1$. To measure *K* to 5% with 1% precision in L_1, *mK* must be at least 0.1. The above conditions are met for the urea data where *K* is 0.04 for concentrations above 2 *m* and for the purine data with $K = 3\ m^{-1}$ above 0.03 *m*.

The feature that emerges in the interpretation of heats of dilution data is the continual presence of the factor $K\Delta H°$. If either factor is known, then heats of dilution measurements provide a highly sensitive method for determining the other term. In cases where reasonable amounts of complex species are formed (as given by a *Km* value of approximately 0.1), it becomes possible to use calorimetric measurements alone to find *K* and then $\Delta H°$.

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